Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes)	Paper Reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3} \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3

Give each coefficient as a fraction in its simplest form.

(6)

Get the formula for the Binomial Expansion from the formula booklet:

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$, Range of validity $|x|<1$

We see that the given expansion is in form $(a+bx)^n$, we need to manipulate to get $(1+ax)^n$.

$(2+5x)^{-3} = (2)^{-3}(1+\frac{5}{2}x)^{-3} = \frac{1}{8}(1+\frac{5}{2}x)^{-3}$ M1B1

Substitute into the Formula:

$\frac{1}{8}(1+\frac{5}{2}x)^{-3} = \frac{1}{8}\left(1+(-3)(\frac{5}{2}x) + \frac{(-3)(-4)}{2!}(\frac{5}{2}x)^2 + \frac{(-3)(-4)(-5)}{3!}(\frac{5}{2}x)^3 + \ldots\right)$

$= \frac{1}{8}\left(1 - \frac{15}{2}x + \frac{\cancel{12}^{3}}{2}(\frac{25}{\cancel{4}_{1}}x^2) + (\frac{\cancel{-60}^{10}}{\cancel{6}})(\frac{125}{8}x^3) + \ldots\right)$

$= \frac{1}{8}\left(1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3 + \ldots\right)$ M1A1

$= \frac{1}{8} - \frac{15}{16}x + \frac{75}{16}x^2 - \frac{625}{32}x^3 + \ldots$ A1A1

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Question 1 continued

Q1

(Total for Question 1 is 6 marks)

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2. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

(a) ☆ Implicit differentiation:

When differentiating y, multiply by $\frac{dy}{dx}$

$$x^3 + 2xy - x - y^3 - 20 = 0$$

Product Rule: $\frac{d}{dx}(uv) = uv' + u'v$

$$3x^2 + 2x\frac{dy}{dx} + 2y - 1 - 3y^2\frac{dy}{dx} = 0$$ M1A1B1

$$3x^2 + 2y - 1 = 3y^2\frac{dy}{dx} - 2x\frac{dy}{dx}$$ collect all $\frac{dy}{dx}$ terms on oneside

$$3x^2 + 2y - 1 = \frac{dy}{dx}(3y^2 - 2x)$$ Factor out $\frac{dy}{dx}$

dM1

$$\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$$ A1 Make $\frac{dy}{dx}$ the subject

(b) Use the $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ we found in (a) to get the gradient at $P(3,-2)$ given

Substitute $P(3,-2)$ into $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}$$

$$= \frac{27 - 4 - 1}{12 - 6}$$

$$\frac{dy}{dx} = \frac{22}{6} = \frac{11}{3}$$ gradient m at P

Use $y - y_1 = m(x - x_1)$ to get the equation of the line:

$m = \frac{11}{3}$, $P(3,-2)$: given

$$y - (-2) = \frac{11}{3}(x - 3)$$ M1

$$3y + 6 = 11x - 33$$

$$0 = 11x - 3y - 39$$ hence found

A1

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Question 2 continued

Q2

(Total for Question 2 is 7 marks)

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3. $$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C

(4)

(b) (i) Hence find $\int f(x)\,dx$

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form $a + \ln b$, where a and b are constants.

(6)

(a) We are asked to do Partial Fractions

$$\frac{1}{x(3x-1)^2} \equiv \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

We realize this is a repeated root, so leave one root squared, the other unsquared

$$\frac{1}{x(3x-1)^2} = \frac{A(3x-1)^2 + Bx(3x-1) + Cx}{x(3x-1)^2}$$

Equate the numerators and substitute in values in order to get A, B and C.

$$1 = A(3x-1)^2 + Bx(3x-1) + Cx \quad \text{B1}$$

choose values to substitute in so that brackets become 0, to make your life easier!

$$x = \tfrac{1}{3} \Rightarrow 1 = A\cancel{(3\times\tfrac{1}{3}-1)}^2 + B(\tfrac{1}{3})\cancel{(3\times\tfrac{1}{3}-1)} + \tfrac{1}{3}C$$

$$1 = \tfrac{1}{3}C \Rightarrow C = 3 \quad \text{A1}$$

$$x = 0 \Rightarrow 1 = A(3(0)-1)^2 + \cancel{B(0)(3(0)-1)} + \cancel{C(0)}$$

$$1 = A \quad \text{M1}$$

$$x = 1 \Rightarrow 1 = (3(1)-1)^2 + B(1)(3(1)-1) + 3(1)$$

$$1 = 4 + 2B + 3 \Rightarrow B = -3$$

Hence:

$$\frac{1}{x(3x-1)^2} \equiv \frac{1}{x} - \frac{3}{(3x-1)} + \frac{3}{(3x-1)^2} \quad \text{A1}$$

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Question 3 continued

(b) i. Use the P.F. Result we got in (a)

$$\int \frac{1}{x(3x-1)^2}\,dx = \int \frac{1}{x} - \frac{3}{(3x-1)} + \frac{3}{(3x-1)^2}\,dx$$

$$= \int \frac{1}{x} - \frac{3}{3x-1} + 3(3x-1)^{-2}\,dx$$

$$= \ln x - 3\ln(3x-1) \times \frac{1}{3} + 3(3x-1)^{-1} \times \frac{1}{3(-1)}$$ divide by new power

M1 Reverse chain Rule: divide by derivative of the bracket

$$= \ln x - \ln(3x-1) - (3x-1)^{-1}$$ A1

$$= \ln x - \ln(3x-1) - \frac{1}{3x-1} + c$$ A1

ii. Implement the given limits on the result we got in (ii.)

$$\int_1^2 \frac{1}{x(3x-1)^2}\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1}\right]_1^2$$

$$= \left(\ln 2 - \ln(3(2)-1) - \frac{1}{3(2)-1}\right) - \left(\overset{0}{\cancel{\ln(1)}} - \ln(3(1)-1) - \frac{1}{3(1)-1}\right)$$ M1

$$= \ln 2 - \ln 5 - \frac{1}{5} + \ln 2 + \frac{1}{2}$$

$$= \ln \frac{2 \times 2}{5} + \frac{3}{10}$$ M1 use ln laws: $\ln a + \ln b = \ln a \times b$

$\ln a - \ln b = \ln \frac{a}{b}$

$$= \frac{3}{10} + \ln \frac{4}{5}$$ A1

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Question 3 continued

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Question 3 continued

Q3

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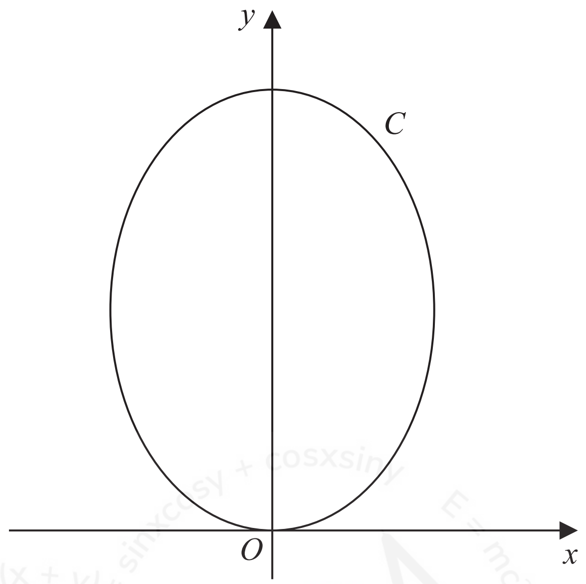

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3}\sin 2t \qquad y = 4\cos^2 t \qquad 0 \leqslant t \leqslant \pi$$

(a) Show that $\dfrac{dy}{dx} = k\sqrt{3}\tan 2t$, where k is a constant to be found.

(5)

(b) Find an equation of the tangent to C at the point where $t = \dfrac{\pi}{3}$

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

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Question 4 continued

(a) ★ Parametric differentiation:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$ get there by differentiating the parametrics separately.

$y = 4\cos^2 t$

$\frac{dy}{dt} = 8\cos t \cdot (-\sin t)$

chain Rule: multiply by derivative of the bracket

$\frac{dy}{dt} = -8\cos t\sin t$ M1 A1

$x = \sqrt{3}\sin 2t$

$\frac{dx}{dt} = 2\sqrt{3}\cos 2t$ B1

$$\frac{dy}{dx} = \frac{-8\cos t\sin t}{2\sqrt{3}\cos 2t}$$

$$= \frac{-4(2\cos t\sin t)}{2\sqrt{3}\cos 2t}$$

apply double angle formula: $\sin 2t = 2\sin t\cos t$

$$= \frac{-4\sin 2t}{2\sqrt{3}\cos 2t}$$ M1

use Identity: $\tan t = \frac{\sin t}{\cos t}$

$$= -\frac{\cancel{4}^{2}}{\cancel{2}\sqrt{3}}\tan 2t$$

$$= -\frac{2\sqrt{3}}{3}\tan 2t$$

$$\frac{dy}{dx} = -\frac{2}{3}\sqrt{3}\tan 2t, \quad k = -\frac{2}{3}$$ A1

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Question 4 continued

(b) Substitute $t=\frac{\pi}{3}$ into the $\frac{dy}{dx}$ we found above to get gradient:

$$\frac{dy}{dx} = -\frac{2}{3}\sqrt{3}\tan\left(2\cdot\frac{\pi}{3}\right)$$

$$= -\frac{2}{3}\sqrt{3}\left(-\sqrt{3}\right)$$

$$\frac{dy}{dx} = 2$$ gradient m at $t=\frac{\pi}{3}$

M1

Get Cartesian Coordinates by using given equations $x=\ldots$ and $y=\ldots$

$x=\sqrt{3}\sin\left(2\times\frac{\pi}{3}\right)$

$x=\sqrt{3}\times\frac{\sqrt{3}}{2}$

$\Rightarrow x=\frac{3}{2}$

$y=4\cos^2\left(\frac{\pi}{3}\right)$

$\Rightarrow y=1$

B1

Hence point where $t=\frac{\pi}{3}$ is $\left(\frac{3}{2},1\right)$.

Use $y-y_1=m(x-x_1)$ to get the equation of the tangent:

$m=2$, $x=\frac{3}{2}$, $y=1$

$y-1=2\left(x-\frac{3}{2}\right)$ dM1

$y=2x-3+1$

$y=2x-2$ found

A1

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Question 4 continued

Q4

(Total for Question 4 is 9 marks)

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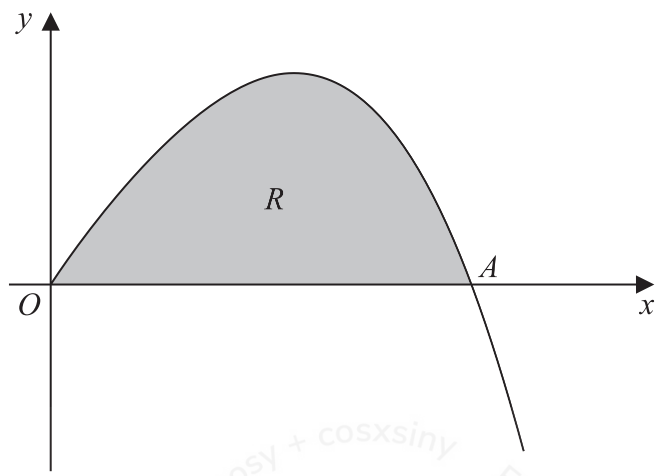

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of $\ln 2$, the x coordinate of the point A.

(2)

(b) Find $\int xe^{\frac{1}{2}x}\,dx$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

(c) Find, by integration, the exact value for the area of R.

Give your answer in terms of $\ln 2$

(3)

(a) Point A is given to be on the x-axis $\therefore$ $y=0$.

$0 = 4x - xe^{\frac{1}{2}x}$

$0 = x(4 - e^{\frac{1}{2}x})$ factor out x

$\therefore x = 0$, $0 = 4 - e^{\frac{1}{2}x}$

$\Rightarrow e^{\frac{1}{2}x} = 4$ M1

$\ln e^{\frac{1}{2}x} = \ln 4$ apply ln on both sides

$\frac{1}{2}x = \ln 2^2$ apply $\ln e^x = x$

$\frac{1}{2}x = 2\ln 2$ apply ln law: $\ln a^b = b\ln a$

$x = 4\ln 2$ x-coordinate of point A.

A1

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Question 5 continued

(b) $\int xe^{\frac{1}{2}x}\,dx$ Multiplication ∴ By Parts:

$\int uv'\,dx = vu - \int u'v\,dx$

$u = x \xrightarrow{\frac{d}{dx}} u' = 1$

$v = 2e^{\frac{1}{2}x} \xleftarrow{\int} v' = e^{\frac{1}{2}x}$

$$\Rightarrow \int xe^{\frac{1}{2}x}\,dx = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\,dx$$ M1A1

$$= 2xe^{\frac{1}{2}x} - \frac{2}{\frac{1}{2}}e^{\frac{1}{2}x}$$

$$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} + c$$ A1

(c) We are asked to find: $\int_0^{4\ln 2} 4x - xe^{\frac{1}{2}x}\,dx$

We already got the integration result $\int xe^{\frac{1}{2}x}\,dx$ in (b)

Hence get $\int 4x\,dx$:

$$\int 4x\,dx = \frac{4}{2}x^2 = 2x^2$$

$$\therefore \int_0^{4\ln 2} 4x - xe^{\frac{1}{2}x}\,dx = \left[2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)\right]_0^{4\ln 2}$$ B1

$$= \left(2(4\ln 2)^2 - 2(4\ln 2)e^{2\ln 2} + 4e^{2\ln 2}\right) - \left(\cancel{2(0)^2} - \cancel{2(0)e^{\frac{1}{2}(0)}} + 4e^{\frac{1}{2}(0)}\right)$$ M1

$$= 2\times 16(\ln 2)^2 - 8\ln(2)e^{\ln 4} + 4e^{\ln 4} - 4$$ apply ln law $b\ln a = \ln a^b$

$$= 32(\ln 2)^2 - 8\ln(2)(4) + 4(4) - 4$$ use $e^{\ln x} = x$

$$= 32(\ln 2)^2 - 32\ln 2 - 12$$

A1

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total for Question 5 is 8 marks)

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6. Prove by contradiction that, if a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$

(4)

Make an Assumption:

"Assume that there exists positive real numbers a, b such that $a+b<2\sqrt{ab}$" B1

We want our assumption to be the opposite of what we are trying to prove.

In the Proof, we are trying to contradict the assumption

Try to solve $a+b<2\sqrt{ab}$:

$$a+b<2\sqrt{ab}$$

$$(a+b)^2<(2\sqrt{ab})^2$$ Square both sides

$$(a+b)^2-(2\sqrt{ab})^2<0$$

$$a^2+2ab+b^2-4ab<0$$ Expand brackets

$$a^2-2ab+b^2<0$$ Collect like terms

Recognize that this is the expansion of $(a-b)^2$

$$\therefore\ (a-b)^2<0$$ M1A1

This is a contradiction since all real squared numbers must be positive.

Hence if a, b are positive real numbers, then $a+b\geq 2\sqrt{ab}$

A1

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Question 6 continued

Q6

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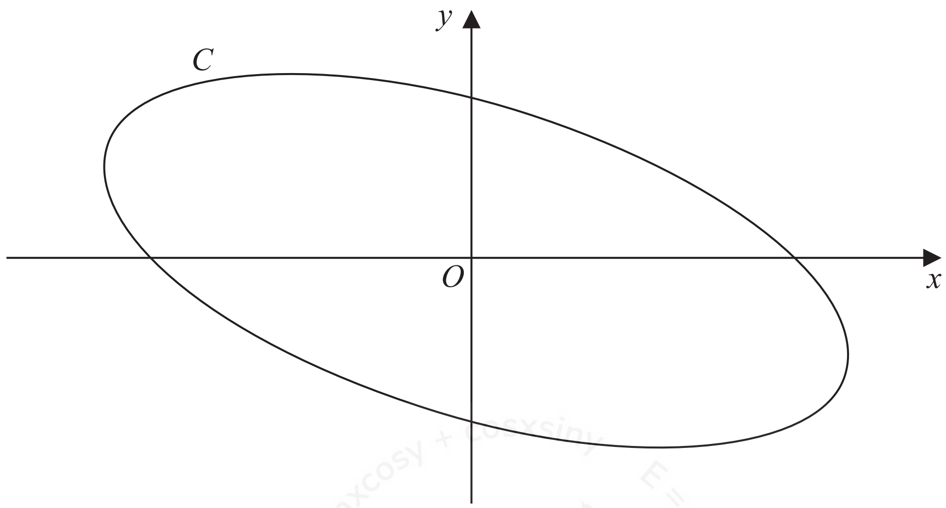

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right) \qquad y = 2\sin t \qquad 0 \leqslant t \leqslant 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be found.

(2)

(a) $x + y = 4\cos(t + \frac{\pi}{6}) + 2\sin t$

$= 4(\cos t\cos\frac{\pi}{6} - \sin t\sin\frac{\pi}{6}) + 2\sin t$ apply addition formula:

M1 $\cos(a+b) = \cos a\cos b - \sin a\sin b$

$= 4\cos t \cdot \frac{\sqrt{3}}{2} - 4\sin t \cdot \frac{1}{2} + 2\sin t$ dM1

$= 2\sqrt{3}\cos t - 2\cancel{\sin t} + 2\cancel{\sin t}$

$x + y = 2\sqrt{3}\cos t$ hence shown

A1

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Question 7 continued

(b) We will use the identity $\sin^2 x + \cos^2 x = 1$ to get the cartesian equation.

Let's get $\sin t = \ldots$ from the given equation:

$y = 2\sin t \Rightarrow \sin t = \frac{y}{2}$

Use what we showed in (a) to get $\cos t = \ldots$:

$x + y = 2\sqrt{3}\cos t \Rightarrow \cos t = \frac{x+y}{2\sqrt{3}}$

Substitute into the identity $\sin^2 x + \cos^2 x = 1$:

$$\left(\frac{y}{2}\right)^2 + \left(\frac{x+y}{2\sqrt{3}}\right)^2 = 1 \quad \text{M1}$$

$$\frac{y^2}{4} + \frac{(x+y)^2}{12} = 1$$

Multiply everything by 12

$$3y^2 + (x+y)^2 = 12$$

$\therefore$ Cartesian: $(x+y)^2 + 3y^2 = 12$ A1

Q7

(Total for Question 7 is 5 marks)

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8. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta) \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

(a) $\frac{d\theta}{dt} = \lambda(120-\theta)$

$\int \frac{1}{120-\theta}\, d\theta = \int \lambda \, dt$ Separate Variables by grouping the like terms on one side B1

$\ln(120-\theta) = \lambda t$ Integrate

-1 Reverse chain rule: divide by derivative of bracket

$-\ln(120-\theta) = \lambda t + c$ M1A1 M1A1

To get value of $+c$ substitute $t=0, \theta=20$:

$-\ln(120-20) = \lambda(0) + c$

$c = -\ln(100)$

Substitute our $+c$ back in and manipulate to make θ the subject:

$-\ln(120-\theta) = \lambda t - \ln(100)$

$\ln(100) - \ln(120-\theta) = \lambda t$ apply ln law $\ln a - \ln b = \ln\frac{a}{b}$

$\ln\left(\frac{100}{120-\theta}\right) = \lambda t$ M1

$e^{\ln\left(\frac{100}{120-\theta}\right)} = e^{\lambda t}$

dddM1 $\frac{100}{120-\theta} = e^{\lambda t}$ use $e^{\ln x} = x$

$\frac{100}{e^{\lambda t}} = 120 - \theta$ cross multiply

$100e^{-\lambda t} - 120 = -\theta$

$\Rightarrow \theta = 120 - 100e^{-\lambda t}$ hence shown A1

Leave blank

Question 8 continued

(b) Substitute $a = 0.01$ and $\theta = 100$, then solve for t:

$$100 = 120 - 100e^{-0.01t}$$ M1

$$100e^{-0.01t} = 20$$

$$e^{-0.01t} = 0.2$$

$$\ln e^{-0.01t} = \ln 0.2$$ use $\ln e^x = x$

$$-0.01t = \ln 0.2$$

$$t = \frac{\ln 0.2}{-0.01}$$ plug this into your calculator

dM1

$\Rightarrow t = 161$ s to the nearest second

A1

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Question 8 continued

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Question 8 continued

Q8

(Total for Question 8 is 11 marks)

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9. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2

(2)

(c) Find the exact value of the distance AP.

Give your answer in the form $k\sqrt{2}$, where k is a constant to be found.

(2)

The acute angle between AP and l_2 is θ

(d) Find the value of $\cos\theta$

(3)

A point E lies on the line l_2

Given that $AP = PE$,

(e) find the area of triangle APE,

(2)

(f) find the coordinates of the two possible positions of E.

(5)

(a) Substitute $\mu=1$ into the given vector equation:

$$A = \begin{pmatrix} 8 \\ 1 \\ 3 \end{pmatrix} + (1)\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 8-5 \\ 1+4 \\ -3+3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}$$

$\therefore$ A(3, 5, 0) B1

Leave blank

Question 9 continued

(b) Formula for vector line:

$r =$ position vector $+ \lambda$(direction vector)

Where the position vector is any point on the line and direction vector is a vector parallel to the line.

The position vector is point $P\begin{pmatrix}1\\5\\2\end{pmatrix}$.

Since l_2 is parallel to l_1, direction vector is that of l_1: $\begin{pmatrix}-5\\4\\3\end{pmatrix}$

$\therefore l_2: r = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ vector line equation

M1A1

(c) Get $\overrightarrow{AP}$: found in (a)

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

$= \begin{pmatrix}1\\5\\2\end{pmatrix} - \begin{pmatrix}3\\5\\0\end{pmatrix}$

$\overrightarrow{AP} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$ M1

Formula for Magnitude of a vector:

Let $\overrightarrow{AB} = \begin{pmatrix}a\\b\\c\end{pmatrix}$. $\Rightarrow$ $|\overrightarrow{AB}| = \sqrt{a^2+b^2+c^2}$

Hence:

$|\overrightarrow{AP}| = \sqrt{2^2+0^2+2^2}$

$= \sqrt{8}$

$= 2\sqrt{2}$ distance AP

A1

(d) Use Formula:

$\cos\theta = \left|\dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}\right|$ Formula for dot product: $\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}d\\e\\f\end{pmatrix} = ad+be+cf$

We will use the vector $\overrightarrow{AP}$ found above and the direction vector of l_2.

$\therefore \cos\theta = \left|\dfrac{\begin{pmatrix}-2\\0\\2\end{pmatrix}\cdot\begin{pmatrix}-5\\4\\3\end{pmatrix}}{2\sqrt{2}\times\sqrt{5^2+4^2+3^2}}\right|$ M1dM1

from (c)

$\cos\theta = \dfrac{10+6}{2\sqrt{2}\times\sqrt{50}}$

$= \dfrac{16}{\sqrt{400}} = \dfrac{16}{20}$

$\therefore \cos\theta = \dfrac{4}{5}$ A1

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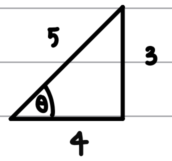
Question 9 continued

(e) We are given that AP = PE, hence $AP = PE = 2\sqrt{2}$ (from (c)).

Hence use formula:

$\text{area} = \frac{1}{2}ab\sin C$

Since $\cos\theta = \frac{4}{5}$ (from (d)):



$\Rightarrow \sin\theta = \frac{3}{5}$.

Use this and $a = b = 2\sqrt{2}$ with the formula:

$$\text{area } APE = \frac{1}{2}(2\sqrt{2})^2 \cdot \frac{3}{5} \quad \text{M1}$$

$$= \frac{12}{5}$$

$$= 2.4 \text{ units}^2 \quad \text{A1}$$

(f) We know that $|\vec{PE}| = 2\sqrt{2}$ (given and part (c)).

We also know that $\vec{PE} = \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ since it lies on l_2.

Hence use formula for magnitude and solve for λ:

Formula for Magnitude of a vector:

let $\vec{AB} = \begin{pmatrix}a\\b\\c\end{pmatrix}$. $\Rightarrow |\vec{AB}| = \sqrt{a^2+b^2+c^2}$

$$|\vec{PE}| = 2\sqrt{2} = \left(\sqrt{(5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2}\right) \quad \text{M1}$$

$$2\sqrt{2} = \sqrt{25\lambda^2 + 16\lambda^2 + 9\lambda^2}$$

$$(2\sqrt{2})^2 = 25\lambda^2 + 16\lambda^2 + 9\lambda^2 \quad \text{Square both sides}$$

$$8 = 50\lambda^2$$

$$\lambda^2 = \frac{4}{25} \Rightarrow \lambda = \pm\frac{2}{5}. \quad \text{A1}$$

$\therefore \vec{OE} = \begin{pmatrix}1\\5\\2\end{pmatrix} \pm \frac{2}{5}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ (using l_2 equation since point E lies on it)

dM1

Hence possible coordinates of E:

$E(3, \frac{17}{5}, \frac{4}{5})$ and $E(-1, \frac{33}{5}, \frac{16}{5})$

A1 A1

Leave blank

Question 9 continued

Q9

(Total for Question 9 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS